EQUILIBRIA IN NETWORKS

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We study a model in which two carriers choose networks to connect cities and compete for customers. We show that if carriers compete aggressively (e.g., Bertrand-like behavior), one carrier operating a single hub-spoke network is an equilibrium outcome. Competing hub-spoke networks are not an equilibrium outcome, although duopoly equilibria in nonhub networks can exist. If carriers do not compete aggressively, an equilibrium with competing hub-spoke networks exists as long as the number of cities is not too small. We provide conditions under which all equilibria consist of hub-spoke networks.

KEYWORDS: Economies of density, hub-spoke networks, competition.

1. INTRODUCTION

THE AIRLINE DEREGULATION ACT of 1978 allowed air carriers to set their own fares and to fly when and where they desire. Carriers responded by transforming their networks into predominantly hub-spoke networks. A large empirical literature has developed which focuses on the impact of hubbing on the supply and prices of air services. A number of papers (e.g., Borenstein (1989, 1990), Reiss and Spiller (1989), Kahn (1993)) seek evidence of market power by relating entry behavior and fares in hub and nonhub markets to measures of hub dominance. Other papers (e.g., Brueckner and Spiller (1991, 1994), Brueckner, Dyer, and Spiller (1992), Caves, Christensen, and Tretheway (1984)) emphasize economies of density and attempt to assess their importance in hub-spoke networks. More recently, Berry (1992) and Berry, Carnall, and Spiller (1996) have estimated the effect of hubbing on costs and on markups.

These empirical studies measure economies of density and price-cost margins in existing airline networks, but they do not explain how these factors have caused hubbing to emerge as the dominant feature. Several types of networks other than hub-spoke networks exhibit economies of density and provide scope for the exercise of market power. As we shall see, in a strategic environment, market power may in fact be higher in nonhub networks. Therefore, it is not obvious when and how the interplay between economies of density and market power leads carriers to choose hub-spoke networks.

Hendricks, Piccione, and Tan (1995) and Starr and Stinchcombe (1992) study network choice in a monopoly environment. It is shown that, when economies of density are present, optimization can lead to hubbing. In Hendricks, Piccione, and Tan (1997a), we studied the extent to which a hub-spoke network is a deterrent to small-scale entry. The focus in this paper is on competition between

two large carriers who are unrestricted in their choices of networks. We investigate the conditions under which hub-spoke networks are equilibria.

Our model is a two-stage game in which two carriers simultaneously choose their networks and then compete for travelers. The carrier offering the shortest path has a competitive advantage because length is costly to carrier and traveler. Economies of density are modeled by assuming that a carrier incurs fixed costs in establishing a network and that these costs exceed the potential profits in a point-to-point[2] network, even if the carrier is a monopolist. Consequently, carriers have to pool travelers with different origin and destination cities on the same flights in order to make a profit.

We consider two kinds of strategic environments. When carriers compete aggressively for customers after choosing their networks (e.g., Bertrand-like behavior), then monopoly is an equilibrium outcome: all city-pair markets are serviced by a single carrier operating a hub-spoke network. There is no equilibrium in which both carriers choose hub-spoke networks, although duopoly equilibria in nonhub networks can exist. When carriers do not price aggressively, a duopoly equilibrium with competing hub-spoke networks exists if the number of cities is not small. We provide conditions under which all equilibria are hub-spoke equilibria.

The theoretical literature on strategic network choice is sparse. To our knowledge, no previous studies have provided a characterization of equilibria in networks. Berechman and Shy (1993) study entry in a model with three cities and show that a hub-spoke network is a more effective barrier to entry than a point-to-point network. Oum, Zhang, and Zhang (1995) examine network choice in a duopoly model with three cities and argue that hub-spoke networks have strategic advantages over point-to-point networks. Zhang (1996) studies a model in which two carriers service the same pair of cities from different hubcities and one of the carriers invades the other carrier's spoke market. In a more general social context with an arbitrary number of individuals, Jackson and Wolinsky (1996) study the efficiency and stability of networks formed by self-interested individuals who choose their direct links to others.

The paper is organized as follows. In Section 2, we introduce the model and the main assumptions. In Section 3, we study the case of network choice when carriers behave like Bertrand competitors. In Section 4, we analyze the non-Bertrand case and present existence and uniqueness results for hub-spoke equilibria. Concluding remarks follow in Section 5.

2. THE MODEL

This section consists of two subsections. In Section 2.1, the formal model is presented as a reduced-form game in which carriers choose networks simultaneously. In Section 2.2, the model is interpreted and the main assumptions are discussed.

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[2] In a point-to-point network every pair of cities is serviced by direct flights.

2.1. *Notation and Assumptions*

There are two carriers, indexed by $i = A, B$, a set $N = \{1, 2, \ldots, n\}$ of $n \geq 3$ distinct cities, and individuals living in each city who wish to travel to other cities in N. For simplicity, we shall assume that all travel is one-way. In what follows, subscripts g, h shall be used to index cities.

A carrier establishes a *direct* connection between city g and city h by providing nonstop flights each way between the two cities. Thus, *one* direct connection connects a city-pair in *both* directions. A *network* is a graph or a set of direct connections. Specifically, a network for carrier i is a function $X^i : N \times N \to \{0, 1\}$ such that $X^i(g, h) = X^i(h, g)$ and

$$X^i(g, h) = \begin{cases} 1 & \text{if there is a direct connection between } g \text{ and } h, \\ & g \neq h, \\ 0 & \text{otherwise.} \end{cases}$$

The *size* of X^i is measured by $m^i = (1/2)\Sigma_{g, h \in N} X^i(g, h)$, the number of direct connections. The empty network is denoted by ϕ.

A sequence of cities $\{n_1, n_2, \ldots, n_{z+1}\}$ is called a *path* if the following conditions hold: (i) $X^i(n_t, n_{t+1} = 1$ for $t = 1, \ldots, z$, (ii) $(n_t, n_{t+1}) \neq (n_s, n_{s+1})$ for $t \neq s$, and (iii) $(n_t, n_{t+1}) \neq (n_{s+1}, n_s)$ for any t and s. Condition (i) states that there is a direct connection between each adjacent pair of cities; conditions (ii) and (iii) state that a direct connection cannot be traveled twice, independently of direction. The *length* of the path is z, the number of direct connections. A path is called a *cycle* if the initial and terminal cities are the same.

Two distinct cities, g and h, are *connected* in X^i if there exists a path $\{n_1, n_2, \ldots, n_{z+1}\}$ such that $n_1 = g$, $n_{z+1} = h$. X^i *spans* city g if $X^i(g, h) = 1$ for some $h \in N$. A network X^i is *complete* if it connects every pair of distinct cities. A network C^i is a component of X^i if it is a maximal, connected subnetwork. Formally, C^i satisfies the following conditions: (i) if $C^i(g, h) = 1$, then $X^i(g, h) = 1$; (ii) if C^i spans cities g and h, then g and h are connected in C^i; and (iii) if C^i spans g and not h, then $X^i(g, h) = 0$.

A network without any cycles is a *forest*. A *tree* is a complete forest. Trees have the property that every pair of cities is connected by a unique path. A network X^i of size m^i, $m^i > 0$, is a *hub-spoke network* if there exists a city h, the *hubcity*, such that $\Sigma_{g \in N} X^i(g, h) = m^i$.

The fixed cost of operating a network is determined by its size. Let $F(m)$ denote the costs of a network of size m. F is assumed to be a strictly increasing, weakly concave function with $F(0)$ equal to zero.

We assume that each carrier's operating profits (revenue minus variable costs) are additively separable across city-pair markets. In each city-pair market, they depend solely on the lengths of the paths chosen by the carriers to transport travelers in that market. Let $\pi(z^i, z^j)$ denote the operating profits that carrier i obtains from a city-pair market in which it offers a path of length z^i and carrier j offers a path of length z^j. If a city-pair is connected only by carrier i, i's profits

are denoted by $\pi(z^i, \infty)$ and j's profits, denoted by $\pi(\infty, z^i)$, are equal to zero. Note that profits are independent of the identity of the carrier. They also incorporate any arbitrage restrictions arising from the ability of travelers to purchase separate tickets for the segments of an indirect connection.

We shall assume that a carrier's profits in a city-pair market are nonincreasing in the length of its path:

(A1) $\pi(z, y) \geq \pi(z + 1, y)$.

The inequality reflects travelers' preferences for shorter paths and transport costs that increase with length. In the presence of multiple paths connecting a pair of cities, Assumption (A1) implies that a carrier cannot gain by using a longer path.

To define the profits of a network, it is convenient to work with a different class of functions. We define a *connection function* $\tau : N \times N \to \{1, 2, \ldots \infty\}$ to be a symmetric $(\tau(g, h) = \tau(h, g))$ mapping which assigns to each city pair (g, h) a positive integer representing path length. Given a connection function τ, its *length correspondence*, Γ, assigns to any integer $z \in \{1, 2, \ldots, \infty\}$ the set of city pairs that are connected in τ by paths of length z. Formally,

$$\Gamma(z) = \{(g, h) | \tau(g, h) = z\}.$$

Let $\#\Gamma(z)$ denote the cardinality of $\Gamma(z)$. For any pair of connection functions (τ^i, τ^j) and τ^i's length correspondence Γ^i, define the expression

$$\Pi(\tau^i, \tau^j) = \sum_{z=1}^{\infty} \sum_{\Gamma^i(z)} \pi(z, \tau^j(g, h)) - F(\#\Gamma^i(1)/2).$$

A connection function is *generated* by a network X^i if it assigns ∞ to each pair of cities not connected in X^i and, to each pair of cities connected in X^i, the length of the *shortest* connecting path. Given a pair of networks (X^A, X^B), network profits to carrier A are defined to be equal to $\Pi(\tau^A, \tau^B)$ where τ^i, $i = A, B$, is the unique connection function generated by X^i. Network profits for carrier B are defined symmetrically. Note that this definition of profits requires carriers to use the shortest path in transporting travelers between cities. For simplicity, we have not modeled the choice of paths for city-pairs with multiple paths. Assumption (A1) ensures that none of the results of this paper are affected by this restriction.

We impose two further restrictions on the city-pair profit functions. Define $\pi_M(z) \equiv \pi(z, \infty)$ to be the profit that a monopoly carrier earns when it services a city-pair market with a path of length z:

(A2-i) $F(n(n - 1)/2) > n(n - 1)\pi_M(1)$;

(A2-ii) $(n - 1)2\pi_M(1) + (n - 1)(n - 2)\pi_M(2) > F(n - 1)$.

Recall that a point-to-point network is a network in which a carrier offers nonstop service in every city-pair market. Condition (A2-i) states that a point-

to-point network is not profitable even if the carrier is a monopolist. Notice that, given (A2-i), the properties of $F(\cdot)$ imply that $2m\pi_M(1)$ is strictly less than $F(m)$ for any positive m. Thus, for a flight to be profitable, it has to carry travelers with different destination or origin cities. Condition (ii) states that a complete hub-spoke network is profitable:

(A3-i) $\pi(z, y) + \pi(y, z) \le \pi_M(\min\{y, z\})$,

(A3-ii) $\pi(z, y) \le \pi_M(z)$ for any y.

Assumption (A3) is a restriction on the division of operating profits. (A3-i) states that the total profits obtained by the two carriers cannot exceed the profits that the carrier with the shortest path would earn if the other carrier did not connect the pair of cities. (A3-ii) states that a carrier cannot earn more than monopoly operating profits.

Unless otherwise stated, we shall assume that, if $\pi_M(z)$ is positive, it is strictly decreasing in z.

2.2. Discussion

Our model makes several simplifying assumptions. First, it is implicit in our definition of profits that carriers cannot share a traveler's itinerary. This restriction rules out interlining, which occurs when travelers switch carriers at a connecting point enroute to their destination. The main reason for ignoring interlining at this stage of the analysis is tractability. We discuss this issue in more detail in Section 4.

Second, we ignore the carrier's choice of direction in allocating its planes. If a carrier offers nonstop service from city h to city g, then it must also offer nonstop service from city g to city h. The interpretation is that the planes fly back and forth between pairs of cities. This restriction on traffic flows is made primarily for tractability, although it can be defended as economically efficient.

Third, we assume that the capacity of the plane flying between a pair of cities is essentially infinite. This makes sense if the (daily) travel demand between city-pairs is low. The following quote from Robert L. Crandall, chairman of American Airlines, which appeared in *American Way* (September, 1992), the airline's magazine, describes the kind of situation that we have in mind:

> On an average day, the typical flight from Albuquerque to DFW carries 123 passengers. Of those, only forty-three are bound for DFW. Two are bound for Atlanta, three for Boston, two for London, and seventy-one for twenty-eight other destinations.

The quote suggests that Crandall regards the number of daily travelers from Albuquerque to each of the thirty-one destinations mentioned in the quote to be too few for American Airlines to offer daily nonstop service between Alburquerque and each of the thirty one cities.

If planes are not capacity constrained, then the size of the fleet depends only upon the number of direct connections or links, and not upon the volume of traffic or design of the network. Hence, fleet costs are part of fixed costs. Fixed

costs also include the costs of station and ground site facilities, ticketing and promotion, and administration. Since these costs may increase less than proportionally with the number of direct connections, $F(m)$ is assumed to be weakly concave.

The factors that vary in proportion to the volume of traffic on a direct connection are stewardesses, meals, and to some extent fuel.[3] The proportionality assumption permits a decomposition of network variable costs in terms of city-pair markets. The variable cost assigned to each city-pair market is proportional to the number of passengers serviced in that market times the length of the path used. These costs will vary with network design. For example, in a linear network of size m, some travelers have to take as many as m flights to reach their destination. By contrast, in a hub-spoke network, no one flies more than two flights. Thus, costs related to volume tend to be much lower in a hub-spoke network than in a linear network, although they are likely to be small relative to fixed costs. A more important factor in network design may be the time costs of travelers.

Finally, we assume no substitutability in demand across city-pair markets. This assumption, together with our linearity assumption on network costs, implies that network profits are additively separable across city-pair markets. It permits an interpretation of the $\pi(\cdot, \cdot)$'s as a reduced form description of the equilibrium of the second stage in which the carriers compete for travelers in city-pair markets given their network choices.

In summary, our model is intended to describe networks of cities where demand between any pair of cities is low and economies of density are essential for profitability. Each plane has to carry connecting passengers to be profitable, and the number of planes is determined primarily by the number of direct connections. Most of the network costs are fixed, and the main cost of path length arises from the traveler's cost of time. The model does not apply to city pairs like New York–Los Angeles where direct traffic volumes are so high that more than one plane is required to service the market. In these cases, the direct traffic is sufficient to exhaust any economies of density.

3. MONOPOLY HUB-SPOKE EQUILIBRIA

In this section, we study the effects of aggressive price competition on network choice. Our focus is on situations in which equilibrium profits to a carrier in a city-pair market can be positive only if it has an advantage over its rival. In our model, path length measures travel costs to both the carrier and the traveler. Thus, a carrier has an advantage only if it offers a shorter path. Formally, this condition can be stated as

(BC) $\pi(z, y) = 0$ if $z \geq y$.

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[3] Fuel consumption depends in part upon weight. Approximately 60% of fuel cost is incurred in takeoff and landing so distance is not as much of a factor as one might expect.

Condition (BC) is likely to be satisfied when the market is not differentiated either horizontally or vertically. The length of a path is a measure of its "quality." Under the assumption that consumers have the same willingness to pay for quality and do not care about the identity of the airline, the second-stage game in each city-pair market is essentially a Bertrand game. In equilibrium, if both carriers offer the same "quality," each carrier will price at marginal cost and earn zero profits. If the carriers offer different "qualities" (i.e., path lengths), then the carrier with the lower "quality" path is priced out of the market and the carrier with the higher "quality" path captures the premium that consumers are willing to pay for the shorter path plus any cost differential. Hence, a carrier can earn positive profits only if it offers a shorter path.

The problem that arises when consumers are sufficiently differentiated in their willingness to pay for "quality" is that the carriers may be able to use length to price discriminate and earn positive profits. For example, suppose business travelers are willing to pay a lot more for a direct flight than vacation travelers. If carriers A and B offer the same quality product, each earns zero profits, which is consistent with condition (BC). But suppose carrier A offers a direct flight and carrier B offers a connecting flight. Then carrier A may find it more profitable to set a high price and sell only to business travelers than try to capture the entire market at a lower price. If so, carrier B can profitably service the vacationers at a lower price, assuming its marginal costs are not too high. Profits fail to satisfy condition (BC). They also fail to satisfy the monotonicity property of Assumption (A1).

Let H_m denote the set of hub-spoke networks of size m. The main result of this section is the following.

THEOREM 1: *Suppose* $(A1)$–$(A3)$ *and* (BC) *hold.*

(a) *No pair* (X^A, X^B) *such that* $X^i \in H_{m^i}$, $0 < m^i \le n - 1$, $i = A, B$, *is an equilibrium.*

(b) *If* $X^A \in H_{n-1}$ *and* $X^B = \phi$, *or* $X^B \in H_{n-1}$ *and* $X^A = \phi$, *then* (X^A, X^B) *is an equilibrium.*

Part (a) of Theorem 1 states that a duopoly equilibrium in hub-spoke networks does not exist. Thus, vigorous competition is not consistent with both carriers operating hub-spoke networks of *any* size. Part (b) states that there is an equilibrium in which one carrier chooses a complete hub-spoke network and the other does not establish any network. In this case, the hub-spoke network deters entry and the industry is a monopoly.

The intuition behind the monopoly result is easily explained. When carrier i chooses a hub-spoke network of size $n - 1$, it services $2(n - 1)$ city-pair markets with direct flights and the remaining $(n - 1)(n - 2)$ city-pair markets with one-stop flights. Carrier j can earn positive profits only in the latter set of markets by offering direct flights. However, it cannot obtain a cost advantage in any associated connecting market. Hence, by condition (BC), its profits on a

direct connection cannot exceed $\pi(1,2)$, which, by Assumptions (A2), (A3), and concavity of $F(\cdot)$, is not sufficient to cover network fixed costs.

The example depicted in Figure 1 illustrates that duopoly equilibria in nonhub networks can exist.

EXAMPLE 1: There are five cities. Carrier A has a network that connects cities 2 and 3 with a one-stop flight through city 1. Carrier B also connects this pair of cities but it offers a path that stops at cities 4 and 5. In addition, it offers nonstop service between cities 1 and 4. The profit terms that are positive are as follows: $\pi_M(1) = \pi(1,3) = 1$, $\pi(1,2) = 0.8$, $\pi_M(2) = \pi(2,3) = 0.5$. All other profit
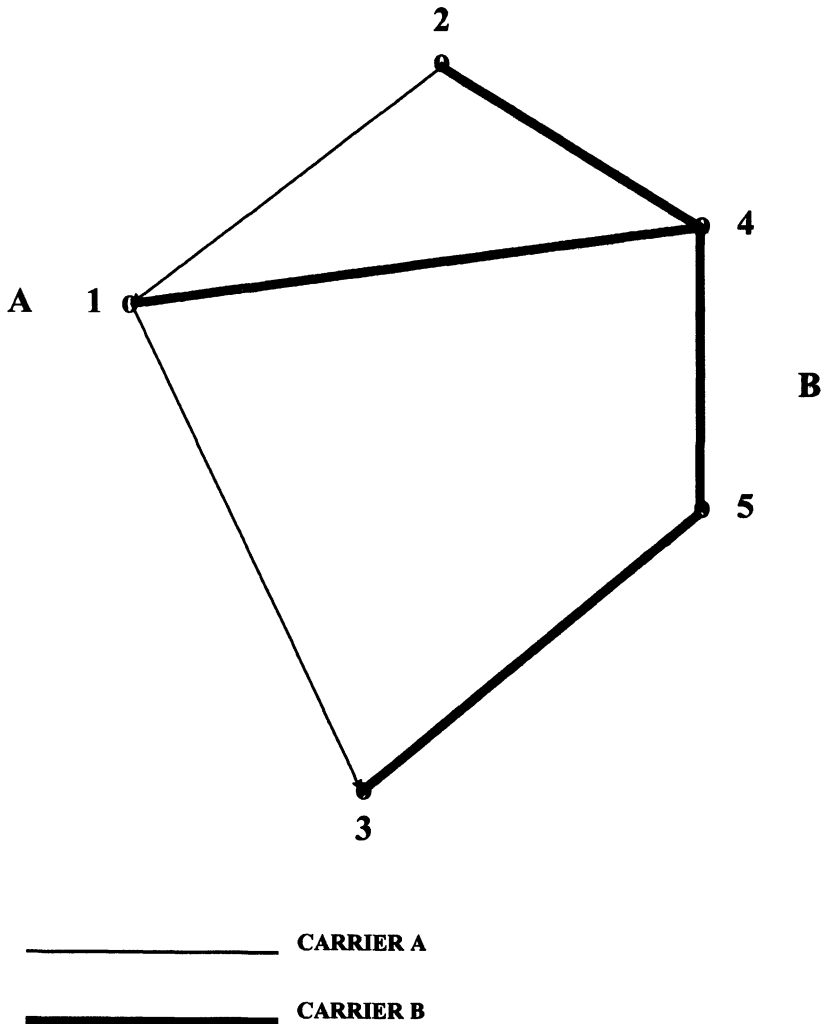

—————————————— CARRIER A

—————————————— CARRIER B

FIGURE 1

terms are zero. Fixed costs are given by $F(m) = 2.2m$. It can be verified that this pair of networks is an equilibrium.[4]

The nonhub configuration is an equilibrium because B is indifferent between servicing the (2–3) market with a path of length 2 or 3; with either path, B obtains zero profits since A is servicing this market with a path of length 2. If B chooses a path of length 2 by hubbing at city 4, then A's best reply is ϕ, the empty network. However, if B chooses the nonhub network with a path of length 3, A's best reply is the one depicted in the figure.

A necessary condition for existence of nonhub duopoly equilibria is that path length is costly to consumers and/or carriers. To make this claim more precise, we state the following theorem.

THEOREM 2: *Suppose* $(A1)$–$(A3)$ *and consider the limit case in which* (a) $\pi(z, y) = 0$ *for* $z, y < \infty$ *and* (b) $\pi(z, \infty) = \bar{\pi}$ *for any* $z < \infty$. *Then* (X^A, X^B) *is an equilibrium if and only if* $X^i = \phi$ *and* X^j *is a tree for* $i \neq j$.

Condition (a) states that if both carriers offer paths connecting two cities, then each carrier earns zero profits in that city-pair market. Condition (b) states that a carrier earns a fixed amount of profit in any city-pair market that it services and its rival does not. Both conditions make sense only if path length is costless (i.e., marginal costs are zero and consumers do not care about path length).

Clearly, under (a) and (b), any pair of networks (X^A, X^B) in which $X^i = \phi$ and X^j is a tree is an equilibrium. Necessity follows from network externalities: if two components in a network are profitable, then combining them will increase profits since it enables the carrier to access with only one additional direct connection a number of new markets equal to the product of the sizes of the components.

The conclusions of Theorem 2 can be sharpened considerably if one considers a small perturbation of the limit case defined by conditions (a) and (b). Since the space of networks is finite, the set of equilibria following a small perturbation must be contained in the set of equilibria in the limit case. However, a carrier is no longer indifferent among all trees. A hub-spoke network yields higher profits since it connects every city pair with a path of length no greater than 2. Hence, a small perturbation selects equilibrium pairs (X^A, X^B) such that $X^i = \phi$ and X^j is a hub-spoke network of size $n - 1$.

4. DUOPOLY HUB-SPOKE EQUILIBRIA

The monopoly result of Theorem 1 provides an interesting benchmark. However, in our view, its relevance is limited. Carriers are likely to find ways to "soften" the competition for customers and avoid marginal cost pricing in

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[4] We are very grateful to Diana Whistler for developing a program to compute best replies to an arbitrary network. Note that the number of possible networks is equal to $2^{n(n-1)/2}$.

city-pair markets where neither has an advantage. The deeper issue is whether hub-spoke networks can be an equilibrium when they are not ruled out on the grounds of profitability. Do duopoly hub-spoke equilibria exist, and perhaps more importantly, are they likely to be the only equilibria?

To address these questions we need to assume that connecting flights are profitable. More precisely, we shall replace condition (ii) of Assumption (A2) by

(A4) $(n-2)(n-3)\pi(2,2) > F(n-2)$.

Assumption (A4) requires profits to be positive in city-pair markets where both carriers offer a one-stop connection. Furthermore, the number of cities reached with a one-stop connection from any city has to be sufficiently large that profits from the associated connecting markets can exceed the fixed costs.

Assumption (A4) will be satisfied if airlines can differentiate their product and build consumer loyalty (e.g., frequent flyer programs). It will also be satisfied in a model where carriers choose quantities rather than prices. It should be noted, however, that quantity competition in city-pair markets cannot be justified in the usual way as a reduced form of a two-stage game in which firms first choose capacity and then price (see Kreps and Scheinkman (1983)). In our model, a direct connection has essentially infinite capacity. Consequently, the quantity game has to be viewed as a reduced-form model of tacit collusion.

In characterizing the set of equilibria under (A1)–(A4), the following property of $\pi(\cdot,\cdot)$ will prove to be important.

DEFINITION: π is *quasi-submodular* if for any pair of positive integers (z,y)

$$\pi(z,y) + \pi(y,z) \geq \pi(z,z) + \pi(y,y),$$

and *quasi-supermodular* if the opposite inequality holds.[5]

To understand why modularity matters, suppose carrier j has a direct connection in one market and a one-stop connection in another market. In the submodular case, carrier i's payoff in these markets is lower if it matches path lengths than if it uses a direct connection against carrier j's one-stop connection and a one-stop connection against carrier j's direct connection. The converse is true in the supermodular case.

Standard models of airline differentiation (assuming consumers do not care about length and marginal costs are constant) under either price or quantity competition generate profit functions that are submodular. The same is true of homogenous or vertically differentiated markets under quantity competition. Supermodularity tends to occur in models where airlines collude explicitly on price (see Appendix B of HPT (1997b)). The essential feature of these models is that carriers have to sell tickets to earn revenues. They cannot collude efficiently by allocating all of the market to the carrier offering the shortest flight and

[5]Actually, what matters for our results is whether π is quasi-submodular or quasi-supermodular on the sublattice, $\Omega = \{(1,1),(1,2),(2,1),(2,2)\}$.

letting that carrier make transfers to the other carrier. The problem, of course, with models of collusion is that it is not clear why the two carriers are not also colluding on networks. In our view, submodularity is the more plausible description.

4.1. *Existence of Hub-Spoke Equilibria*

We show first that, if a duopoly is viable, carrier i's best response to a hub-spoke network of size $n - 1$ is a hub-spoke network. The relative locations of the two hubcities and the sizes of the hub-spoke networks depend upon the properties of the profit function.

LEMMA 1: *Suppose $(A1)$–$(A4)$ hold. If the set of best replies to a complete hub-spoke network with hubcity h does not contain a complete hub-spoke network, then it contains a hub-spoke network of size $n - 2$ that does not span h.*

The argument consists of two steps. We show first that, if carrier i chooses a complete hub-spoke network, then carrier j's profits from an arbitrary network with fewer than $n - 1$ direct connections can be bounded by the profits of a hub-spoke network of size $n - 1$ or $n - 2$. When this bound is achieved by a hub-spoke network of size $n - 1$, the relative locations of the hubcities are determined by the modularity properties of the profit function. In the second step, we show that the same networks can be used to bound the profits of arbitrary networks of size greater than $n - 1$.

The best reply to a hub-spoke network of size $n - 2$ may or may not be a hub-spoke network. Additional restrictions on the profit function are needed to ensure existence of hub-spoke equilibria. One approach is to require that the best reply to a hub-spoke network of size $n - 1$ is a hub-spoke network of size $n - 1$. Define

$$F_1 \equiv 2\pi(1,1) + 2(n-2)\pi(2,1).$$

THEOREM 3: *Suppose $(A1)$–$(A4)$ hold and $F(n-1) - F(n-2) < F_1$.*
(a) *If π is quasi-supermodular on Ω, $X^i \in H_{n-1} = A, B$, and $X^A = X^B$, then (X^A, X^B) is an equilibrium.*
(b) *If π is quasi-submodular on Ω, $X^i \in H_{n-1}$, $i = A, B$, and $X^A \neq X^B$, then (X^A, X^B) is an equilibrium.*

The condition of the Theorem states that a carrier must be able to earn positive profits in city-pair markets in which it offers a connecting flight and its rival offers a direct flight. In the submodular case, it implies that a carrier will want to establish a direct connection between the two hubcities, assuming n is sufficiently large. In the supermodular case, it implies that using the same complete hub-spoke network as its rival is more profitable than any network of size $n - 2$.

4.2. *Existence of Nonhub Equilibria*

One may be tempted to argue that a sufficient condition for all equilibria to be hub-spoke equilibria is that connections longer than two are not profitable. Under this restriction, if a carrier wants to service every city-pair market profitably with a nonhub network, it would have to choose a larger, more costly network. The following example, depicted in Figure 2, illustrates that the situation is considerably more complex.

EXAMPLE 2: There are six cities. Carrier A's network directly connects the following city pairs: $(1,4)$, $(1,5)$, $(1,6)$, and $(3,5)$. Carrier B's network directly connects city pairs $(1,2)$, $(2,3)$, $(3,4)$, and $(3,6)$. Neither network is complete. The positive profit terms are as follows: $\pi(1,z) = 1$ for $z \geq 3$, $\pi(1,1) = 0.2$, $\pi(1,2) = 0.4$, $\pi(2,z) = 0.5$ for $z \geq 4$, $\pi(2,1) = 0.01$, $\pi(2,2) = 0.15$, $\pi(2,3) = 0.2$. All other profit terms are zero. Note that the only connections that are profitable are of length less than 3. Fixed costs are given by $F(m) = 2.2m$. It can be verified that each carrier's network is the unique best reply.
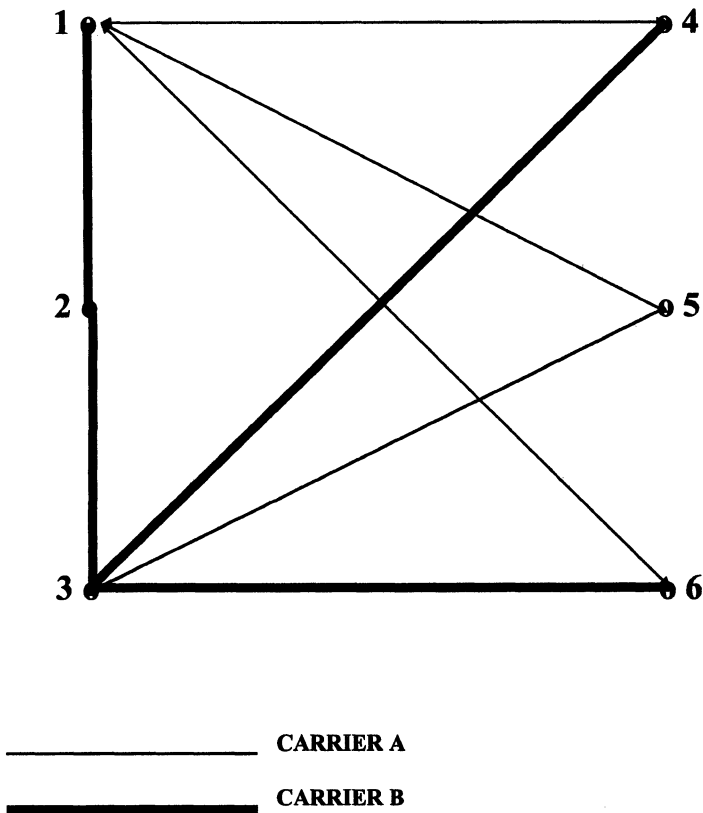

CARRIER A

CARRIER B

FIGURE 2

Hubbing allows carriers to service more city-pair markets with a one-stop flight. In Example 2, if carrier A hubs at city 1 by dropping the (3–5) direct connection and adding a direct connection between cities 1 and 3, it can service the (4–3) and (3–4) markets with a one-stop connection rather than a two-stop connection. However, the gains from this transformation may be small relative to decrease in monopoly profits associated with the (3–5) and (5–3) markets, which have gone from $\pi_M(1)$ to $\pi_M(2)$. Intuitively, carrier A is better off sacrificing a couple of indirect markets by locating its direct flight to capture a market not serviced by carrier B. Hubbing at other cities involves similar tradeoffs.

It is worth noting that the nonhub duopoly equilibrium in Example 2 is not the only equilibrium. Hub-spoke equilibria in which each carrier chooses four direct connections centered in distinct hubcities also exist. However, these hub-spoke equilibria are less profitable. Profits to each carrier are equal to 1.0 in the hub-spoke equilibria and to 1.8 in the above nonhub equilibrium.

We shall show that the following condition rules out nonhub equilibria:

(A5-i) $\pi(1, z) - \pi(2, z) \le \inf_s \{\pi(2, s) - \pi(3, s)\}$ for any z.

(A5-ii) $\pi(1, z) > \pi(2, z)$ for any z.

Assumption (A5-i) penalizes paths of length 3 or more relative to shorter paths and imposes an upper bound on the difference in profits between direct and one-stop connections. More precisely, it states that the difference in profits between a direct and a one-stop connection is small relative to the difference between a one-stop and a two-stop connection, independently of the path lengths offered by a rival. Condition (ii) states that direct flights are always more profitable than one-stop flights.

THEOREM 4: *Suppose $(A1)$–$(A5)$ hold, $n > 5$ and $F(m) = mf$. If (X^A, X^B) is an equilibrium, then X^i, $i = A, B$ is a hub-spoke network.*[6]

The proof of Theorem 4 is quite involved and we break it up into a series of lemmas. We first consider networks that have $n - 1$ or more direct connections and show that a hub-spoke network of size $n - 1$ is the most profitable of these networks, independently of the rival's network. Lemma 2 presents a technical result that may be of independent interest to graph theorists. Define L to be the number of city-pairs in a network that are connected only by paths of length equal to $3, 4, \ldots, \infty$. The number of cities that are directly connected to a city g is known as the *degree* of g. Let D denote the maximum degree achieved in the network.

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[6]This theorem holds for $n \le 5$ as well. The proof for this case is different and involves tedious circulations. It is omitted.

LEMMA 2: *Consider a network of size m and suppose that $m \geq n - 1$ and $n > 5$. Then $L \geq \max\{0, 2(2(n - 1) - m - D)\}$.*

Lemma 2 establishes a lower bound on the number of city-pairs that are connected only by paths of length greater than 2 in any network of size $n - 1$ or larger. For example, a network containing a hub-spoke subnetwork of size $n - 1$ has a city with degree $n - 1$, which implies a nonpositive value for the lower bound. Since L is zero in such a network, the inequality is satisfied. In a circle network, the degree of each city in a circle network is 2, so the lower bound is $2(n - 4)$. It is easily checked that the number of city-pairs connected by paths of length 3 or more is $n(n - 5)$. Note that, when n is equal to 5, all city-pairs are connected by paths of length 2 or less and the bound fails.

Lemma 3 establishes the dominance of hub-spoke networks of size $n - 1$ in the subset of networks with $n - 1$ or more direct connections.

LEMMA 3: *Suppose $(A1)$–$(A5)$ hold, $n > 5$, and $F(m) = mf$. If a network X^i of size m^i is a best reply and $m^i \geq n - 1$, then $X^i \in H_{n-1}$.*

The proof reflects the effects of economies of density and declining profitability with path length. We first bound the profits of a candidate network by a connection function that has $m - n - 1$ fewer direct connections but is otherwise identical to the candidate network. In dropping these direct connections, we ignore the impact on connecting markets and work with a connection function that does not necessarily represent a network. This step requires that the loss in operating profits not exceed the reduction in fixed costs from dropping a direct connection. The linearity of $F(m)$ combined with Assumption $(A2)$ ensures that this condition is always satisfied.[7] Lastly, we apply Lemma 2 and Assumption $(A5)$ to show that the profits of a hub-spoke network are an upper bound on the profits of the connection function that is obtained from the candidate network by dropping direct connections.

The next step in the proof involves evaluating networks that are smaller than $n - 1$.

LEMMA 4: *Suppose $(A1)$–$(A5)$ hold and $F(m) = mf$. If a network X^i of size m^i is a best reply and $m^i \leq n - 1$, then each component of X^i is a hub-spoke network.*

Lemma 4 establishes that any network that has fewer than $n - 1$ direct connections must consist of a collection of hub-spoke components. The intuition for ruling out nonhub components is similar to that of the previous lemma.

The remaining step of the proof involves showing that each carrier prefers a single hub-spoke network to a collection of hub-spoke networks. The reason is network externalities: if each hub-spoke component is profitable, then combin-

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[7]An alternative assumption under which Lemma 3 and Theorem 4 hold is that $F(m) - F(m - 1)$ exceeds $2\pi_M(1)$ for every m.

ing them increases profits. Several cases need to be considered depending upon the locations of the hubcities.

REMARK 1: In the context of airline networks, Assumption (A5) should be interpreted as a restriction on consumer preferences. It requires travelers to marginally prefer direct to one-stop connections but to strongly prefer one-stop to multi-stop connections. Consequently, it is easily satisfied (together with Assumption (A4)) in models of vertical differentiation, provided carriers compete in quantities.[8] To determine the empirical relevance of Assumption (A5), it would be useful to have measures of the premiums that travelers are willing to pay for fewer stop-overs.

Another industry where Assumption (A5) is plausible is express cargo services. Since Federal Express established the first hub-spoke network in 1973, its major competitors, UPS, DHL, and the U.S. Postal Services, have also established hub-spoke network operations based in, respectively, Cincinnati, Indianapolis, and Louisville. In this industry, consumers have strong preferences for next day delivery. Their demands may exclude connections with more than one stop.

REMARK 2: Theorem 4 rules out equilibria in networks other than hub-spoke networks, but makes no statement about their size. It is not hard to find examples in which the equilibria involve incomplete hub-spoke networks of varying sizes. Additional restrictions on π and F as in Theorem 3 are required to ensure completeness.

Also, pure strategy equilibria may not exist. For example, suppose the number of cities is 4, $F(m) = 2.2m$, and profits are as follows: $\pi(1, 1) = 0.5$, $\pi(1, z) = 1$ for $z \geq 2$, $\pi(2, 1) = 0$, $\pi(2, 2) = 0.25$, $\pi(2, z) = 0.5$ for $z \geq 3$. All other terms are equal to zero. It can be verified that there is no pure strategy equilibrium in this example.

REMARK 3: We have assumed throughout this paper that interlining is not allowed. Interlining is essentially a bargaining problem. One issue that arises is the timing of the negotiations, that is, whether carriers agree to interlining routes simultaneously with or after they choose their networks. The other issue is the specification of equilibrium profits in city-pair markets serviced by interlining paths. On the first issue, we believe that the appropriate approach is to assume that interlining negotiations occur after networks are chosen. Thus, carriers choose networks noncooperatively but are allowed to negotiate profit shares in specific city-pair markets. Of course, carriers will anticipate the

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[8]For example, suppose consumer θ's utility from purchasing a trip of length z at price p is given by $U = V(z)\theta - p$, where $V(z)$ is a decreasing function. His utility if he does not purchase a trip is zero. Here θ is distributed according to some distribution F and marginal costs are assumed to be zero. It is easily verified that the profit functions (under quantity competition) are submodular and satisfy Assumptions (A1)–(A4). Furthermore, if $V(2)$ does not differ much from $V(1)$ and $V(z)$ is small for $z \geq 3$, then (A5) is also satisfied.

outcomes of such negotiations in their choice of network. In particular, paths that lie in a carrier's network can be used by that carrier as credible outside options to interlining routes.

We intend to study the problem of interlining in more detail in a subsequent paper. Our preliminary results suggest that interlining affects network profitability only if (i) they connect cities that are otherwise not connected or (ii) they are shorter than the paths offered in the carriers' own networks. Neither situation is present when one of the carriers has a complete hub-spoke network.[9] Consequently, hub-spoke equilibria remain equilibria (Theorems 1(b), 2, and 3) when interlining is permitted conditional on network choice. However, interlining does affect the profitability of incomplete and nonhub networks, which raises the possibility of additional equilibria.

5. CONCLUSIONS

Aggressive competition leads to a monopoly outcome if one of the carriers chooses a complete hub-spoke network. However, duopoly equilibria can exist. In Example 1, carriers are able to position their networks so that there are no markets in which both carriers offer the same path length. Every city-pair market is effectively serviced by only one carrier, the one with a length advantage. The local monopoly power conferred by this advantage must be present in some connecting markets for duopolies to exist in a Bertrand-type, competitive environment. A complete hub-spoke network does not allow for this possibility and it is in this sense that it deters entry. Furthermore, this monopoly outcome is the only equilibrium if the gain from a length advantage is small.

When carriers earn profits in city-pair markets where neither has a length advantage, then duopoly equilibria in hub-spoke networks can exist. However, contrary to views expressed in the literature, strategic play does not appear to be the most crucial factor in the selection of hub-spoke networks. In our model, a monopolist always chooses a hub-spoke network whereas duopolists may not. Strategic factors determine whether a carrier will try to match or mismatch the lengths of its connections against those of its rivals. A carrier can realize this objective with a hub-spoke network if its rival network is also a hub-spoke. However, the same objective can lead to a nonhub network if the rival's network is nonhub. Furthermore, as Example 2 shows, hub-spoke equilibria can be Pareto-dominated (from the carriers' viewpoint) by nonhub equilibria. Nonhub networks raise average costs of service but may allow the carriers to price less aggressively.

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[9] In the United States, prior to deregulation in 1978, entry restrictions caused airline networks to be incomplete and nonhubs. The only path between many pairs of cities often consisted of several direct connections offered by different carriers. However, after deregulation, hubbing emerged and interlining traffic as a share of connecting traffic fell from 38.8% in 1979 to 4.5% in 1989 (Bamberger and Carleton (1993)).

In the U.S., airlines operate networks that appear to be predominantly hub-spoke. However, they contain more than one hubcity and subnetworks that are point-to-point operations. The former may reflect distance factors[10] that lead carriers to operate regional hub-spoke networks. The latter, which gives the network a two-tier structure, can be explained as a failure of symmetry in demand and Assumption (A2). The volume of direct traffic between some city-pairs may be sufficiently large to profitably support a direct connection. Further work is needed to explore the theoretical implications of relaxing these assumptions.

The larger empirical question that this paper raises is the significance of departures from a single hub-spoke network. If significant, the issue is whether they are the outcome of strategic interaction, as shown in Examples 1 and 2, or of factors excluded from our model such as demand asymmetries or distance and scheduling constraints. If not significant, then our model suggests that pricing is not Bertrand.

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APPENDIX

We begin with a lemma that is not stated in the text but is fundamental to the results. The proof is a minor modification of Theorem 2 in Hendricks, Piccione, and Tan (1995) and is not given.

LEMMA 0: *Suppose (A2) holds. Then the optimal monopoly network is a hub-spoke network.*

PROOF OF THEOREM 1: (a) Suppose that (X^A, X^B) is an equilibrium and that X^A and X^B are hub-spoke networks. Let N^i denote the set of cities spanned by X^i, and n^i the number of cities in N^i, $i =$ A,B. Also, let q be the number of cities in $N^A \cap N^B$. Thus, the number of city pairs connected by both A and B is $q(q-1)$. Also, if $q = n^i$, (BC) implies that carrier i's city-pair profits can be positive only for directly connected city-pair markets. Hence, by (A2) and the properties of $F(\cdot)$, $q \leq \min\{n^A - 1, n^B - 1\}$.

Suppose first that either A and B have the same hubcity or the hubcity of carrier A is not in N^B. Since A's direct connections are never matched with B's indirect connections, the profits of carrier A can be bounded by

(P.1) $\qquad (n^A - 1)^2 \pi_M(1) + ((n^A - 1)(n^A - 2) - q(q-1)) \pi_M(2) - F(n^A - 1).$

[10] A referee has pointed out an intriguing analogy between our model in which all cities are equally distant from each other and the Dixit-Stiglitz model in which all products are equally distant from each other.

Note that this network profit can be achieved by carrier A with a hub-spoke network with a hubcity not in N^B. Hence, our claim follows if we show that if A adds one direct connection to a city in N^B, its network profits increase. Doing so, A's network profits are

(P.2) $n^A 2\pi_M(1) + (n^A(n^A - 1) - (q + 1)q)\pi_M(2) - F(n^A)$.

Subtracting (P.1) from (P.2) yields

(P.3) $2\pi_M(1) + 2(n^A - 1 - q)\pi_M(2) - (F(n^A) - F(n^A - 1))$.

The properties of F imply that (P.3) is positive if

$$2\pi_M(1) + 2(n^A - 1 - q)\pi_M(2) - F(n^A - 1)/(n^A - 1) > 0.$$

By (A2), $\pi_M(2) > 0$. Since (P.1) is nonnegative, the claim is proved if

(P.4) $((n^A - 1)(n^A - 2) - q(q - 1))/(n^A - 1) < 2(n^A - 1 - q)$.

Note that (A2) and the properties of $F(\cdot)$ imply that (P.1) is negative when $q \geq n^A - 1$. Hence, $q < n^A - 1$. Our claim then follows since (P.4) simplifies to $(n^A - q) - (n^A - q)^2 < 0$.

Suppose now that carrier A's hubcity is in N^B but different from carrier B's hubcity. First note that if carrier B's hubcity is not in N^A, repeating the above step for carrier B would suffice. Thus, assume that carrier A directly connects the two hubcities. This implies that $q \geq 2$. Since $\pi(1,1) = 0$, the network profits of carrier A are

$$(n^A - q)^2 \pi_M(1) + (q - 2)^2 \pi(1,2)$$
$$+ ((n^A - 1)(n^A - 2) - q(q - 1) + 2(q - 1))\pi_M(2) - F(n^A - 1)$$

where the term multiplying $\pi_M(2)$ is obtained noting that, of $q(q - 1)$ city pairs connected by both carriers, $2(q - 1)$ are directly connected by carrier A. If $\pi(1,2) = 0$, (P.1) is again a bound on A's network profits and the claim can be proved as above. Suppose that $\pi(1,2) > 0$. Then, by dropping the direct connection between the hubcities and adding a direct connection from carrier A's hubcity to a nonhubcity in N^B, carrier A increases profits. Since $q \leq n^B - 1$, such transformation is possible and the claim follows.

(b) Suppose X^i is a hub-spoke network of size $n - 1$. Since carrier i connects every pair of cities with a path whose length is either one or two, (BC) implies that carrier j's profits from any network X^j of size $m^j > 0$ are bounded by $m^j 2\pi(1,2) - F(m^j) \leq m^j 2\pi_M(1) - F(m^j) < 0$, where the inequalities follows from (A3) and (A2). Hence, carrier j is best off not connecting any cities. It follows from (A2) and Lemma 0 that the optimal network for carrier i is a hub-spoke network of size $n - 1$.   Q.E.D.

PROOF OF THEOREM 2: Conditions (a) and (b) imply that X^i contains no cycles. By dropping one direct connection in the cycle, the number of connected city-pairs is unchanged. Hence, fixed costs are reduced and operating profits are unaffected.

Suppose X^A and X^B are forests but not trees. Recall that a component of X^A, denoted by C^A, is a maximal connected subnetwork of X^A. Given a component C^A, let V^A denote the set of cities spanned by C^A and v^A the number of cities in V^A. Select an arbitrary city $g \notin V^A$ and let t be the number of cities in V^A connected to g in X^B (possibly zero). Carrier B connects at least $t(t - 1)$ pairs of cities in V^A. Let m^A be the size of X^A and consider

(P.5) $((v^A(v^A - 1) - t(t - 1))\bar{\pi} - (v^A - 1)F(m^A)/m^A$.

Since X^A is an equilibrium network, (P.5) is nonnegative for at least one component. Letting C^A be that component, we have that $t < v^A - 1$ since $2m^A \bar{\pi} < F(m^A)$.

Now suppose carrier A connects city g to a city in V^A. Then, its gain in network profits is, by the properties of $F(\cdot)$, at least

(P.6) $2(v^A - t)\bar{\pi} - F(m^A)/m^A$.

Multiplying (P.6) by $(v^A - 1)$ and manipulating the expression, we obtain

(P.7) $2(v^A - 1)(v^A - t)\overline{\pi}(v^A - 1)F(m^A)/m^A$

$$= (v^A - t)(v^A - t - 1)\overline{\pi} + (v^A(v^A - 1) - t(t - 1))\overline{\pi} - (v^A - 1)F(m^A)/m^A.$$

Since (P.5) is nonnegative and $t < v^A - 1$, (P.7) is positive. Hence, X^A must be a tree. It then follows from (a) that carrier B cannot earn any city-pair profits since all city-pairs are connected in X^A. B's best response is then the empty network. Therefore, under (a) and (b), (X^A, X^B) is an equilibrium if and only if X^i is a tree and $X^j = \phi$. $\hspace{2cm}$ Q.E.D.

PROOF OF LEMMA 1: Let X^B be a hub-spoke network of size $n - 1$ and consider the best reply for carrier A. (A4) implies that the empty network is not a best reply since a hub-spoke network of size $n - 2$ which does not span the hubcity of X^B yields positive profits. Let X^A be a best reply network with m^A direct connections, and let s denote the number of cities directly connected to the hubcity of X^B in both networks. We proceed in several steps.

The first step is to show that the profits of X^A with $m^A \le n - 2$ can be bounded by the profits of a hub-spoke network of size $n - 2$ or size $n - 1$.

Suppose that $s \ge 1$. In this case, the network profits of carrier A from the pair (X^A, X^B), denoted by Π^A, can be bounded as follows:

$$\Pi^A \le 2s\pi(1, 1) + 2(m^A - s)\pi(1, 2) + 2(m^A - s)\pi(2, 1)$$
$$+ [m^A(m^A - 1) - 2(m^A - s)]\pi(2, 2)F(m^A)$$
$$= m^A[2\pi(1, 2) + 2\pi(2, 1) - 2\pi(2, 2) + (m^A - 1)\pi(2, 2)]$$
$$+ 2s[\pi(1, 1) + \pi(2, 2) - \pi(1, 2) - \pi(2, 1)] - F(m^A).$$

The above inequality is obtained as follows. First, (A1) allows us to substitute $\pi(2, \cdot)$ for any $\pi(z, \cdot)$, $z > 2$. Second, consider the component of X^A that contains carrier B's hubcity and suppose that this component has m' direct connections and spans k cities. By Hendricks, Piccione, and Tan (1995, Lemma 1), the maximum number of connected city pairs in the component is $m'(m' + 1)$. Using the definition of a component, it then follows that $k \le m' + 1$. Applying Lemma 1 again, the maximum number of city-pairs connected in X^A is $k(k - 1) + (m^A - m')(m^A - m' + 1)$. Of these, $2m^A$ are directly connected by carrier A (i.e., $\pi(1, 1)$ and $\pi(1, 2)$ terms). The number of nonhubcities indirectly connected by A to B's hub is $(k - 1 - s)$, so the number of $\pi(2, 1)$ terms is $2(k - 1 - s)$. The number of $\pi(2, 2)$ terms is then bounded by $[k(k - 1) + (m^A - m')(m^A - m' + 1) - 2m^A - 2(k - 1 - s)]$. This last expression is smaller than $[m^A(m^A - 1) - 2(m^A - s)]$ for $m' \le m^A$.

If π is quasi-submodular, $s = 1$ determines an upper bound. By (A4), $\pi(2, 2)$ is positive. Since Π^A is positive, the properties of $F(\cdot)$ imply that setting $m^A = n - 1$ determines again an upper bound. Hence,

$$\Pi^A \le 2\pi(1, 1) + 2(n - 2)\pi(1, 2) + 2(n - 2)\pi(2, 1)$$
$$+ (n - 2)(n - 3)\pi(2, 2) - F(n - 1).$$

The bound given in the last line can be achieved by a hub-spoke network of size $n - 1$ located at a nonhubcity in X^B.

If π is quasi-supermodular, then set $s = m^A$. Applying the same argument as above, one can show that Π^A is bounded by the profits of a hub-spoke network identical to X^B.

Now suppose that $s = 0$. Then carrier A does not service any of the markets directly connected by carrier B. The network profits of carrier A from the pair (X^A, X^B) can then be bounded as follows:

$$\Pi^A \le m^A[2\pi(1, 2) + (m^A - 1)\pi(2, 2)] - F(m^A).$$

Since the right-hand side of the above inequality is positive, applying the usual argument one can show that Π^A is bounded by the profits of a hub-spoke network of size $n - 2$ that does not span the hubcity of X^B. This concludes the first step.

The next step of the proof consists of showing that the profits of a network X^A with $m^A \geq n-1$ are also bounded by the profits of a hub-spoke network of size $n-1$ or $n-2$. Suppose first that $s \geq 1$. (A1) implies that

$$\Pi^A \leq 2s\pi(1,1) + 2(m^A - s)\pi(1,2) + 2(n-1-s)\pi(2,1)$$
$$+ [n(n-1) - 2m^A - 2(n-1-s)]\pi(2,2) - F(m^A)$$
$$= 2s[\pi(1,1) - \pi(1,2) - \pi(2,1) + \pi(2,2)] + m^A[2\pi(1,2) - 2\pi(2,2)]$$
$$+ 2(n-1)\pi(2,1) + (n-1)(n-2)\pi(2,2) - F(m^A).$$

If $\pi(z^i, z^j)$ is quasi-submodular, we obtain

$$\Pi^A \leq 2[\pi(1,1) - \pi(1,2) - \pi(2,1) + \pi(2,2)] + m^A[2\pi(1,2) - 2\pi(2,2)]$$
$$+ 2(n-1)\pi(2,1) + (n-1)(n-2)\pi(2,2) - F(m^A).$$

Denote the right-hand side of the above inequality by $\Lambda^*(m^A)$ and note that $\Lambda^*(n-1)$ is achieved by a hub-spoke network of size $n-1$ centered in a nonhubcity in X^B. Suppose that $\Lambda^*(m^A) > \Lambda(n-1)$ and $\Lambda^*(m^A) \geq 0$. Let $\overline{m}$ be the smallest m^A for which $\Lambda^*(m^A) > \Lambda^*(n-1)$ and $m^A > n-1$. Then, $\Lambda^*(\overline{m}) > \Lambda^*(\overline{m} - 1)$, which simplifies to $2\pi(1,2) - 2\pi(2,2) > F(m) - F(m-1)$. Since the right-hand side of the above inequality is nonincreasing in $\overline{m}$, $\Lambda^*(m^A)$ is strictly increasing for $m^A \geq \overline{m}$. Thus, $\Lambda^*(m^A) < \Lambda^*(n(n-1)/2)$ and, by quasi-submodularity,

$$\Lambda^*(n(n-1)/2) \leq [n(n-1)/2][2\pi(1,2) - 2\pi(2,2)] + 2(n-1)\pi(2,1)$$
$$+ (n-1)(n-2)\pi(2,2) - F(n(n-1)/2)$$
$$= (n-1)(n-2)\pi(1,2) + 2(n-1)[\pi(1,2) + \pi(2,1) - \pi(2,2)]$$
$$- F(n(n-1)/2).$$

Then, by (A3-i) and (A2), $\Lambda^*(n(n-1)/2) \leq n(n-1)\pi_M(1) - F(n(n-1)/2) < 0$. A contradiction. If $\pi(z^i, z^j)$ is quasi-supermodular,

$$\Pi^A 2(n-1)[\pi(1,1) - \pi(1,2) - \pi(2,1) + \pi(2,2)] + m^A[2\pi(1,2) - 2\pi(2,2)]$$
$$+ 2(n-1)\pi(2,1) + (n-1)(n-2)\pi(2,2) - F(m^A).$$

Using the same argument as in the submodular case, one gets again a contradiction.

Suppose now $s = 0$. Then carrier A does not connect any markets directly connected by carrier B and X^A does not span B's hubcity. Hence,

$$\Pi^A \leq m^A[2\pi(1,2) - 2\pi(2,2)] + (n-1)(n-2)\pi(2,2) - F(m^A).$$

Proceeding as in the previous two cases, one can show that (A2) and (A3-i) imply that an upper bound is attained when $m^A = n-2$. This bound is achieved by a hub-spoke network of size $n-2$ not spanning the hubcity of X^B.   Q.E.D.

PROOF OF THEOREM 3: Let X^B be a hub-spoke network of size $n-1$. It follows from Lemma 1 that carrier A's best reply can be restricted to the set of hub-spoke networks of size $n-1$ and the set of hub-spoke networks of size $n-2$ not spanning the hubcity of X^B. The claim then follows by comparing the profits of these networks.   Q.E.D.

PROOF OF LEMMA 2: Suppose first that there exists a city h' that is directly connected to at most one city, say h, and let d_h denote the degree of h. Then

$$L \geq 2[(n-1) - d_h] \geq 2[2(n-1) - m - d_h].$$

Suppose next that every city has a degree higher than one. Since $\Sigma_{h \in N} d_h = 2m$, it follows that $m \geq n$. If the city with maximum degree has degree strictly greater than $2(n-1) - m - 1$, then the claim is satisfied since $L > 0$.

Recall that D is the maximum degree and suppose that $D \leq 2(n-1) - m - 1$. Note that (i) the number of city-pairs connected by paths of length two is bounded from above by $\sum_{h \in N} d_h(d_h - 1)$; and (ii) the number of city-pairs connected by paths of length one is $\sum_{h \in N} d_h$. Thus, the number of city-pairs connected by paths of length two or less, $n(n-1) - L$, is bounded from above by $\sum_{h \in N}(d_h)^2$. Fixing m and D, consider the following maximization problem:

$$\text{choose } \{x_h\}_{h \in N} \text{ to maximize } \sum_{h \in N}(x_h)^2 \text{ subject to } 2 \leq x_h \leq D \text{ and } \sum_{h \in N} x_n = 2m.$$

Since the objective function is convex, the solution admits at most one x_h in the interior of the first constraint. The other x_h's must be equal to either 2 or D. Let λ be the integer that solves $2(n - \lambda) + d + (\lambda - 1)D = 2m$, for some d such that $2 \leq d \leq D$.

Note that if $D = 2$, then $m = n$ and $\sum_{h \in N}(d_h)^2 = 4n$. Hence, $L \geq n(n-1) - 4n$. Our claim follows if $n(n-1) - 4n \geq 2[2(n-1) - n - 2]$, which holds for $n \geq 6$.

Next, suppose $D > 2$. The maximized value of the above objective is

$$V = 4(n - \lambda) + d^2 + (\lambda - 1)D^2$$
$$\leq 4(n - \lambda + (D - d)/(D - 2)) + D^2(\lambda + 1 + (d - 2)/(D - 2))$$
$$= 4(n - k) + kD^2$$

where k solves $2(n - k) + kD = 2m$. The inequality follows noting that

$$4(D - d)/(D - 2) + D^2(d - 2)/(D - 2) - d^2 = (D - d)(d - 2) \geq 0$$

and the equality follows by relabelling since $d = 2(D - d)/(D - 2) + D(d - 2)/(D - 2)$. Then $L \geq n(n-1) - 4(n - k) - kD^2$. Therefore, our claim follows if

$$n(n-1) - 4(n - k) - kD^2 - 2(2(n-1) - m - D) \geq 0$$

for $2 < D \leq 2(n-1) - m - 1$. Substituting for k, the inequality simplifies to

$$4 + 2D(1 - m + n) - 2m - 5n + n_2 \geq 0.$$

Since $D > 2$, it follows that $m \geq n + 1$ and the left-hand side is nonincreasing in D. Setting $D = 2(n-1) - m - 1$, we get

(P.8) $-2 + 2m + 2m^2 - 7n + 6mn + 5n^2 \geq 0.$

The left-hand side is minimized at $m = 3n/2 - 1/2$. Thus, the above inequality holds if $n_2 - 8n - 5 \geq 0$, which holds for $n \geq 9$. The reader can verify that (P.8) holds for the cases $n = 6, 7, 8$ for $m \geq n + 1$ and $D > 2$.   Q.E.D.

PROOF OF LEMMA 3: Without loss of generality, we fix B's network and consider the best reply for carrier A. The proof is by contradiction. Suppose that X^A is a best reply to X^B. Let τ^i denote the connection function generated by X^i and h^* be a city spanned by X^A whose degree achieves the maximum degree D^A. Let L^A denote the number of city-pairs in X^A that are connected only by paths of length strictly greater than 2.

Case 1: Suppose that $D^A = n - 1$. If X^A is not a hub-spoke network, then $m^A > n - 1$. It then follows from (A2) that dropping direct connections between city-pairs (g, h) such that $g, h \neq h^*$ increases network profits. Thus, a hub-spoke network centered in city h yields higher profits than X^A.

Case 2: Suppose next that $D^A < n - 1$ and $m^A \geq [2(n-1) - D^A]$. Network profits from X^A are given by

$$\Pi(\tau^A, \tau^B) = \sum_{z=1}^{\infty} \sum_{(g,h) \in \Gamma^A(z)} \pi(z, \tau^B(g, h)) - m^A f.$$

These profits can be bounded as follows.

(i) For every $(g, h) \in \Gamma^A(z)$ and $z > 2$, replace $\pi(z, \tau^B(g, h))$ with $\pi(2, \tau^B(g, h))$.

(ii) Select $2(m^A - (n - 1))$ city-pairs $(g, h) \in \Gamma^A(1)$ such that $g, h \neq h^*$. For $2[(n - 1) - D^A]$ of these city-pairs, replace $\pi(1, \tau^B(g, h))$ with $\pi(3, \tau^B(g, h))$. For the remaining $2[m^A - 2(n - 1) + D^A]$ city-pairs (which is nonnegative by assumption) replace $\pi(1, \tau^B(g, h))$ with $\pi(2, \tau^B(g, h))$.

(iii) Drop $m^A - (n - 1)f$ terms.

(A1) ensures that Step 1 does not lead to a decrease in network profits.

(A2) implies that Step 2 combined with Step 3 increases network profits since $m^A > n - 1$. Let $\tilde{\tau}$ denote the connection function obtained from τ^A by Steps 1–2 and $\tilde{\Gamma}$ its length correspondence. We have that

(P.9) $$\Pi(\tau^A, \tau^B) < \Pi(\tilde{\tau}, \tau^B)$$
$$= \sum_{(g, h) \in \tilde{\Gamma}(1)} \pi(1, \tau^B(g, h)) + \sum_{(g, h) \in \tilde{\Gamma}(2)} \pi(2, \tau^B(g, h))$$
$$+ \sum_{(g, h) \in \tilde{\Gamma}(3)} \pi(3, \tau^B(g, h)) - (n - 1)f.$$

Note that, letting $\#$ denote the cardinality of a set,

$$\#\tilde{\Gamma}(1) = 2(n - 1), \quad \#\tilde{\Gamma}(2) = n(n - 1) - 2(2(n - 1) - D^A), \quad \#\tilde{\Gamma}(3) = 2(n - 1 - D^A).$$

Now consider the profits of a hub-spoke network centered in city h^* and denote its connection function by τ^*. We bound these profits from below by replacing $\pi(1, \tau^B(g, h^*))$ with $\pi(2, \tau^B(g, h^*))$ in each city-pair market (g, h^*) such that $\tilde{\tau}(g, h^*) \neq 1$. The number of such markets is $2(n - 1 - D^A)$. Let $\hat{\tau}$ denote the connection function associated with this transformation and $\hat{\Gamma}$ its length correspondence. By (A5-ii) and using the fact that $D^A < n - 1$, we obtain a strict lower bound for the profits of τ^*:

(P.10) $$\Pi(\tau^*, \tau^B) > \Pi(\hat{\tau}, \tau^B)$$
$$= \sum_{(g, h) \in \hat{\Gamma}(1)} \pi(1, \tau^B(g, h)) + \sum_{(g, h) \in \hat{\Gamma}(2)} \pi(2, \tau^B(g, h)) - (n - 1)f.$$

Define $S = \{(g, h) | \tilde{\tau}(g, h) = 1$ and $\hat{\tau}(g, h) = 2\}$ to be the set of city pairs that are assigned a 1 by $\tilde{\tau}$ but not by $\hat{\tau}$. Note that $\#S = 2(n - 1 - D^A)$. Subtracting the bound in (P.9) from the one in (P.10) then yields

$$\Pi(\tilde{\tau}, \tau^B) - \Pi(\hat{\tau}, \tau^B) = \sum_{(g, h) \in S} [\pi(1, \tau^B(g, h)) - \pi(2, \tau^B(g, h))]$$
$$+ \sum_{(g, h) \in \tilde{\Gamma}(3)} [\pi(3, \tau^B(g, h)) - \pi(2, \tau^B(g, h))] \leq 0.$$

The inequality follows from the fact that $\#S = \#\tilde{\Gamma}(3)$ and (A5). The claim then follows by (P.9) and (P.10).

Case 3: Finally, suppose $D^A < n - 1$ and $m^A [2(n - 1) - D^A]$. Profits from τ^A need to be bounded as follows:

(i) For every $(g, h) \in \Gamma^A(z)$ and $z \geq 3$, replace $\pi(z, \tau^B(g, h))$ with $\pi(3, \tau^B(g, h))$.

(ii) Having done the transformation in Step 1, select $L^A - 2[2(n - 1) - m^A - D^A]$ city-pairs with $\pi(3, \tau^B(g, h))$ and replace them with $\pi(2, \tau^B(g, h))$. By Lemma 2, this transformation is feasible if $n > 5$.

(iii) Select $2(m^A - (n - 1))$ city-pairs $(g, h) \in \Gamma^A(1)$ such that $g, h \neq h^*$. For each city-pair, replace $\pi(1, \tau^B(g, h))$ with $\pi(3, \tau^B(g, h))$.

(iv) Drop $m^A - (n - 1)f$ terms.

Let $\bar{\tau}$ denote the connection function generated from τ^A by Steps 1–3. By construction, the cardinality of the sets $\bar{\Gamma}(1)$, $\bar{\Gamma}(2)$, and $\bar{\Gamma}(3)$ induced by $\bar{\tau}$ is the same as the sets $\tilde{\Gamma}(1)$, $\tilde{\Gamma}(2)$, and $\tilde{\Gamma}(3)$ in Case 2. The rest of the proof for this case proceeds in exactly the same manner as in Case 2.   Q.E.D.

PROOF OF LEMMA 4: Once again, we fix B's network and prove the claim for carrier A. Recall that a component of X^A is a maximal connected subnetwork of X^A. Select a component C^A and let V^A denote the set of cities spanned by C^A. The number of cities in V^A will be denoted by v^A and the number of direct connections in C^A is given by r^A. By definition, $r^A \geq v^A - 1$.

Note first that, if $r^A v^A - 1$, there exists a city that is directly connected to at most one city, say h, with degree d_h. Then, as in the proof of Lemma 2, the number of city-pairs connected by paths of length strictly greater than two is at least $2[2(v^A - 1) - r^A - d_h]$. A straightforward adaptation of the argument given in Case 3 of Lemma 3 shows that C^A must be hub-spoke.

Suppose then that $r^A \geq v^A$. Since $m^A \leq n - 1$, there exists a city h not in V^A. Consider a network obtained by modifying X^A as follows: (i) connect h city directly to each city in V^A; (ii) drop all of the direct connections in C^A. The transformation in (i) adds to A's network profits at least $v^A(2 \min_z\{\pi(1, z)\} - f)$ and transformation in (ii) subtracts at most

$$v^A(2 \max_z\{\pi(1, z) - \pi(2, z)\} - f) + (r^A - v^A)(2 \max_z\{\pi(1, z) - \pi(2, z)\} - f),$$

where min and max are taken over the set of path lengths in carrier B's network. By (A1) and (A2), $(r^A - v^A)(2 \max_z\{\pi(1, z) - \pi(2, z)\} - f) \leq 0$. By (A5), $\min_z\{\pi(1, z)\} > \max_z\{\pi(1, z) - \pi(2, z)\}$. Hence, profits increase. Q.E.D.

PROOF OF THEOREM 4: By Lemmas 3 and 4, any nonempty equilibrium network, X^i, $i = A, B$, is a collection of hub-spoke components. (A4) implies that ϕ is never a best-reply to a hub-spoke network or to itself and Lemma 0 implies that the best reply to ϕ is a complete hub-spoke network. Therefore, $X^i \neq \phi$, $i = A, B$. We will now show that it is always profitable for carriers to combine hub-spoke components into a single hub.

Case 1 (Shared hubcity): Suppose that the equilibrium networks, X^A and X^B, have hub-spoke components, C^A and C^B, with the same hubcity, labelled as h_c. Let r^i denote the number of direct connections in C^i and V^i the set of cities spanned by C^i. Let s denote the number of cities that are directly connected to h_c in C^A and C^B. We will show that a shared hubcity is consistent with equilibrium only if $r^i = s = n - 1$, that is, C^A and C^B are identical, complete hub-spoke networks.

Consider first the subcase in which $s = 0$ and $r^A + r^B < n - 1$. Then there is a city $h \notin V^A \cup V^B$. Note that h must be connected in some other component of X^A to a city g in V^B different from h_c, as depicted in the top panel of Figure 3. If not, directly connecting h to h_c, carrier B gains $2\pi_M(1) + r^B 2\pi_M(2) - f$. It can be easily verified that this expression is positive as the network profits from C^B are nonnegative. Consider a network obtained by modifying X^A as follows: (i) connect directly all of the nonhubcities in V^A to city g; (ii) drop all of the direct connections in C^A. By this transformation carrier A obtains the network depicted in the bottom panel of Figure 3. The reader can verify that: (i) all city-pair markets connected in C^A by paths of length equal to two are still connected by paths of the same length; (ii) the same number of $\tau_M(1)$'s as in C^A is retained since, by definition of components and the hypothesis that $s = 0$, g is not connected by either A or B to the cities in V^A. In addition, the carrier obtains $\pi(2, \cdot)$ terms from connecting each of the nonhubcities in V^A to cities directly connected to g in X^A. By (A5), these terms are positive. Hence, if $s = 0$, then $r^A + r^B = n - 1$.

We show next that if $s = 0$, then $r^i = m^i$. Suppose, on the contrary, that $m^A > r^A$. Then, X^A must span nonhubcities in V^B with a component other than C^A. Label one such city as g and then repeat the transformation described above. The claim then follows. Hence, $s = 0$ implies $C^i = X^i$.

Finally, we show that $s = 0$ cannot be an equilibrium. By the previous steps, carrier A's network profits from X^A are

(P.11) $\Pi^A = m^A[2\pi_M(1) + (m^A - 1)\pi_M(2) - f] \geq 0$.

Carrier A can realize the same profits with a hub-spoke network that spans the same set of cities as X^A but has a different hubcity, say city g. With this network, if A adds a direct connection between city g and a nonhubcity in B's network, then its network profits increase by

$$2\pi_M(1) + (m^A - 1)2\pi_M(2) + 2\pi(2, 1) - f.$$

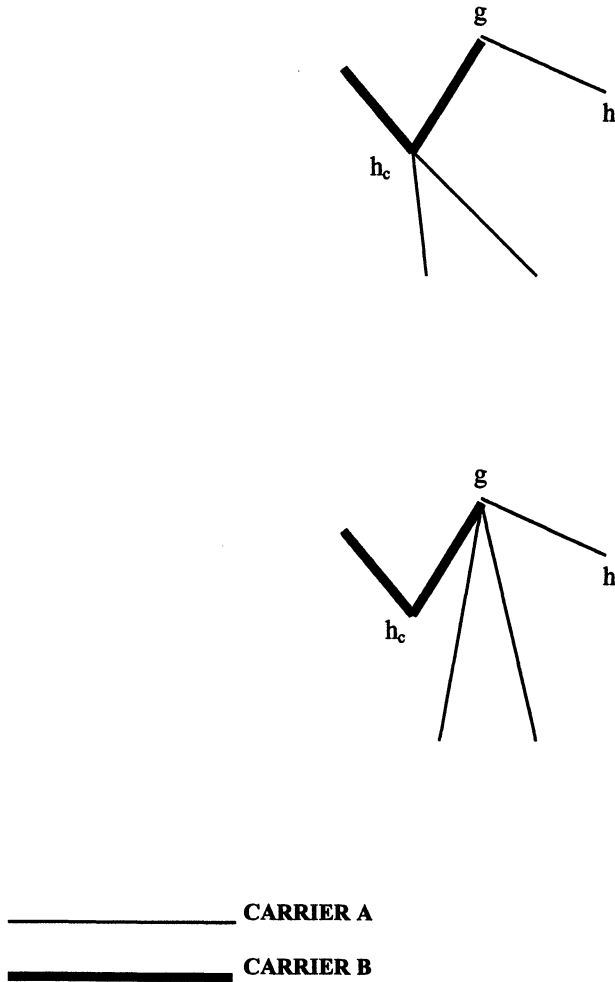(P.11) implies that this gain is positive. Hence, $s = 0$ cannot be an equilibrium.

CARRIER A

CARRIER B

FIGURE 3

We now consider the subcase in which $s > 0$. Select a city $g \in V^A \cap V^B$, $g \neq h_c$. By definition of hub-spoke components, g is directly connected only to h_c. Profitability of this connection implies that

(P.12) $2\pi(1,1) + (s-1)2\pi(2,2) + (r^i - s)2\pi_M(2) - f \geq 0$

for each $i = A, B$. If carrier i adds a direct connection between h_c and a city g' such that $g' \in V^j$ and $g' \notin V^i$, $i \neq j$, its net gain is

$$2\pi(1,1) + s2\pi(2,2) + (r^i - s)2\pi_M(2) - f.$$

It follows from (A4) and (P.12) that the gain is positive. Thus, $s = r^A = r^B$.

Finally, suppose that $r^i < n - 1$. Then there exists a city g that is not connected to any city in the component. It is easy to show that profitability of components implies that each carrier i can gain by connecting city g to h_c. Hence, if $s > 0$, then $s = r^i = n - 1$ and $X^A = X^B \in H_{n-1}$.

Case 2 (Disjoint hubcities): In this case, none of the components of the equilibrium networks X^A and X^B share a hubcity. We show that X^i must be single hub-spoke component. Suppose carrier A has two hub-spoke components, C_1^A and C_2^A, spanning respectively the sets of cities V_1^A and V_2^A, with hubcities h_1^A and h_2^A. We consider several subcases.

(ii-a) Consider a component of X^B, C^B, spanning a set V^B, with hubcity h^B and suppose that h_1^A and h_2^A are elements of V^B and $h_1^A, h_2^A \neq h^B$. The top panel of Figure 4 illustrates this situation. Without loss of generality, designate h_1^A as a hubcity that is not directly connected to h^B. By dropping the direct connections in C_1^A and directly connecting nonhubcities of V_1^A to h_2^A, carrier A obtains the network depicted in the bottom panel of Figure 4. It services all of the city-pairs indirectly connected in C_1^A using h_2^A rather than h_1^A. Thus, it loses no $\pi(2, \cdot)$ terms with this

_____ CARRIER A

━━━━━━━━━━━ CARRIER B
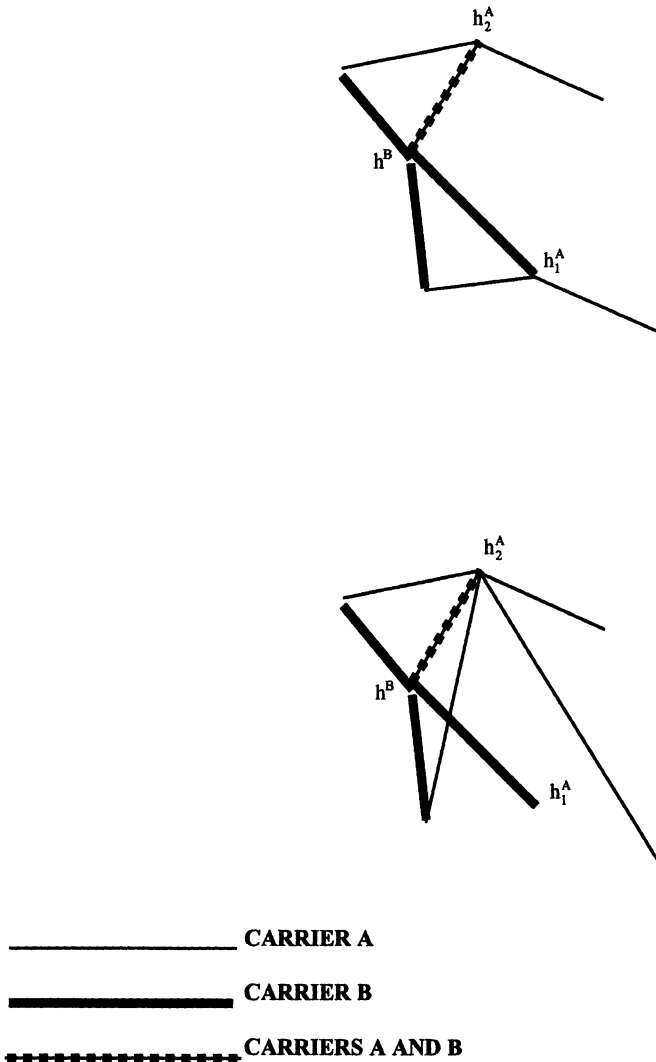
▪▪▪▪▪▪▪▪▪▪▪▪▪▪ CARRIERS A AND B

FIGURE 4

transformation. Since C^B and C_1^A are components, A retains the same numbers of $\pi_M(1)$ and $\pi(1,2)$ terms although these are obtained in different city-pair markets. However, A gains additional $\pi(2,\cdot)s$ by connecting cities in V_1^A to cities in V_2^A. These are positive by (A5).

(ii-b) Consider next two components of X^B, C_1^B and C_2^B, spanning sets V_1^B and V_2^B with hubcities h_1^B and h_2^B respectively, and where $h_1^A \in V_1^B$, $h_2^A \in V_2^B$, $h_1^A \neq h_1^B$, and $h_2^A \neq h_2^B$. We will show that connecting h_1^A and h_2^A yields carrier A higher profits. Suppose first that h_1^A is not directly connected to any city in V_2^B except possibly for h_2^B as depicted in the top panel of Figure 5. Then the profits A obtains from C_1^A are bounded by

(P.13) $r_1[2\pi_M(1) + (r_1 - 1)\pi_M(2) - f]$,

where r_1 is the number of direct connections in C_1^A. By connecting h_1^A to h_2^A, A gains $2\pi_M(1) + (r_1 - \xi)2\pi_M(2) + \xi 2\pi(2,1) - f$, where $\xi = 1$ if $h_2^B \in V_1^A$ and $\xi = 0$ otherwise. Since (P.13) is nonnegative by hypothesis and $\pi(2,1)$ is positive by (A5), the gain is positive, a contradiction.
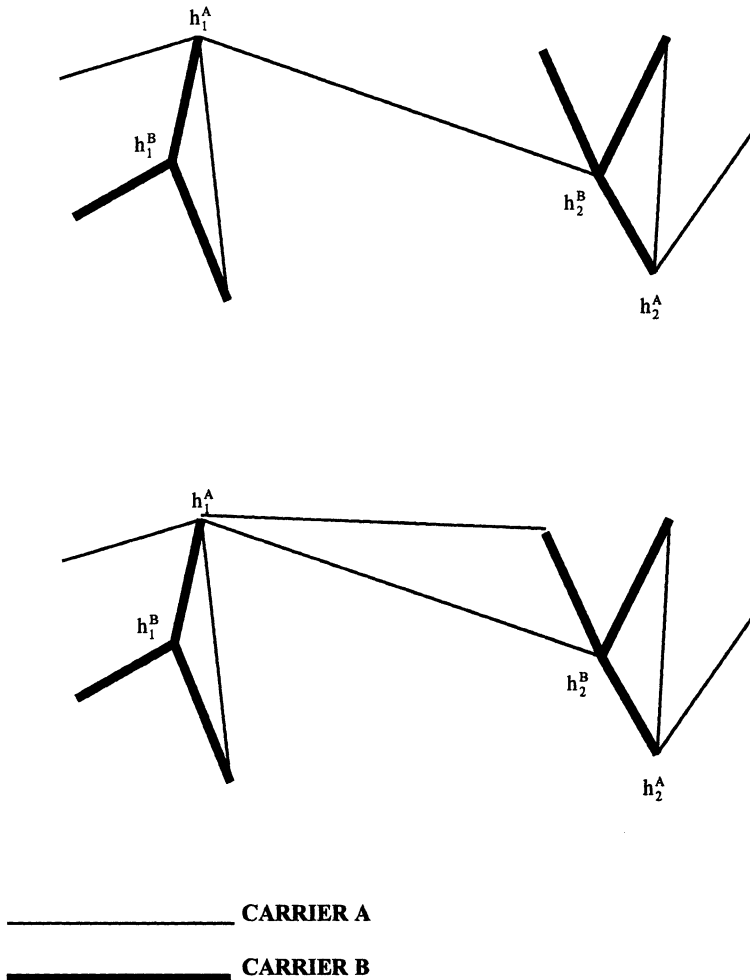

FIGURE 5

Suppose next that h_1^A is directly connected to at least one city in V_2^B different from h_2^B as depicted in the bottom panel of Figure 5. Let k_t denote the number of cities in $V_1^A \cap V^B$, $t = 1, 2$, and let k_3 be the number of cities in V_1^A that are not in $V_1^B \cup V_2^B$. Directly connecting h_1^A and h_2^A yields A at least

$$2\pi_M(1) + (k_1 - 1 + k_3)2\pi_M(2) + (k_2 - \xi)2\pi(2,2) + \xi 2\pi(2,1) - f$$

where $\xi = 1$ if $h_2^B \in V_1^A$ and $\xi = 0$ otherwise. The profitability of a direct connection between h_1^A and a city g in V_2^B different from h^B implies

$$2\pi_M(1) + (k_1 - 1 + k_3)2\pi_M(2) + (k_2 - \xi - 1)2\pi(2,2) + \xi 2\pi(2,1) - f \geq 0.$$

Hence, the claim follows since, by (A4), $\pi(2,2)$ is positive.

(ii-c) Lastly, suppose h_2^A is not spanned by X^B. Then it is easily verified that the transformation outlined in subcase (ii-a) increases profits.   *Q.E.D.*

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